2 Pin-jointed frames or trusses

2.1 Introduction

In problems of stress analysis we discriminate between two types of structure; in the first, the forces in the structure can be determined by considering only its statical equilibrium. Such a structure is said to be *statically determinate.* The second type of structure is said to be *statically indeterminate.* In the case of the latter type of structure, the forces in the structure cannot be obtained by considerations of statical equilibrium alone. This is because there are more unknown forces than there are simultaneous equations obtained from considerations of statical equilibrium alone. For statically indeterminate structures, other methods have to be used to obtain the additional number of the required simultaneous equations; one such method is to consider compatibility, as was adopted in Chapter 1. In this chapter, we will consider statically determinate frames and one simple statically indeterminate frame.

Figure 2.1 shows a rigid beam BD supported by two vertical wires BF and DG; the beam carries a force of $4W$ at C. We suppose the wires extend by negligibly small amounts, so that the geometrical configuration of the structure is practically unaffected; then for equilibrium the forces in the wires must be $3W$ in BF and W in DG. As the forces in the wires are known, it is a simple matter to calculate their extensions and hence to determine the displacement of any point of the beam. The calculation of the forces in the wires and structure of Figure 2.1 is said to be *statically determinate.* If, however, the rigid beam be supported by three wires, with an additional wire, say, between H and J in Figure 2.1, then the forces in the three wires cannot be solved by considering statical equilibrium alone; this gives a second type of stress analysis problem, which is discussed more fully in Section 2.5; such a structure is *statically indeterminate.*

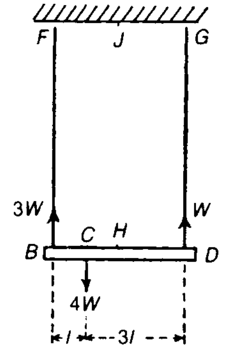


Figure 2.1 Statically determinate system of a beam supported by two wires.

2.2 Statically determinate pin-jointed frames

By a *frame* we mean a structure which is composed of straight bars joined together at their ends. A *pin-jointed frame or truss* is one in which no bending actions can be transmitted from one bar to another as described in the introductory chapter; ideally this could be achieved if the bars were joined together through pin-joints. If the frame has just sufficient bars or rods to prevent collapse without the application of external forces, it is said to be *simply-stiff*; when there are more bars or rods than this, the frame is said to be *redundant*. A redundant framework is said to contain one or more redundant members, where the latter are not required for the framework to be classified as a framework, as distinct from being a mechanism. It should be emphasised, however, that if a redundant member is removed from the framework, the stresses in the remaining members of the framework may become so large that the framework collapses. A redundant member of a framework does not necessarily have a zero internal force in it. Definite relations exist which must be satisfied by the numbers of bars and joints if a frame is said to be simply-stiff, or statically determinate.

In the plane frame of Figure 2.2, BC is one member. To locate the joint D relative to BC requires two members, namely, BD and CD; to locate another joint F requires two further members, namely, CF and DF. Obviously, for each new joint of the frame, two new members are required. If m be the total number of members, including BC, and j is the total number of joints, we must have

$$m = 2j - 3, \tag{2.1}$$

if the frame is to be simply-stiff or statically determinate.

When the frame is rigidly attached to a wall, say at B and C, BC is not part of the frame as such, and equation (2.1) becomes, omitting member BC, and joints B and C,

$$m = 2j \tag{2.2}$$

These conditions must be satisfied, but they may not necessarily ensure that the frame is simply-stiff. For example, the frames of Figures 2.2 and 2.3 have the same numbers of members and joints; the frame of Figure 2.2 is simply-stiff. The frame of Figure 2.3 is *not* simply-stiff, since a mechanism can be formed with pivots at D, G, J, F. Thus, although a frame having j joints must have at least $(2j - 3)$ members, the mode of arrangement of these members is important.

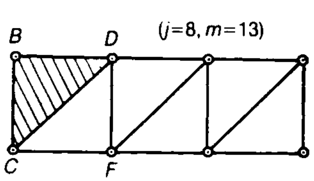


Figure 2.2 Simply-stiff plane frame built up from a basic triangle BCD.

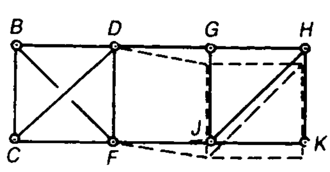


Figure 2.3 Rearrangement of the members of Figure 2.2 to give a mechanism.

For a pin-jointed space frame attached to three joints in a rigid wall, the condition for the frame to be simply-stiff is

$$m = 3j \tag{2.3}$$

where m is the total number of members, and j is the total number of joints, exclusive of the three joints in the rigid wall. When a space frame is not rigidly attached to a wall, the condition becomes

$$m = 3j - 6, \tag{2.4}$$

where m is the total number of members in the frame, and j the total number of joints.

2.3 The method of joints

This method can only be used to determine the internal forces in the members of statically determinate pin-jointed trusses. It consists of isolating each joint of the framework in the form of a *free-body diagram* and then by considering equilibrium at each of these joints, the forces in the members of the framework can be determined. Initially, all *unknown forces* in the members of the framework are assumed to be in *tension*, and before analysing each joint it should be ensured that each joint does not have more than two unknown forces.

To demonstrate the method, the following example will be considered.

Problem 2.1 Using the method of joints, determine the member forces of the plane pin-jointed truss of Figure 2.4.

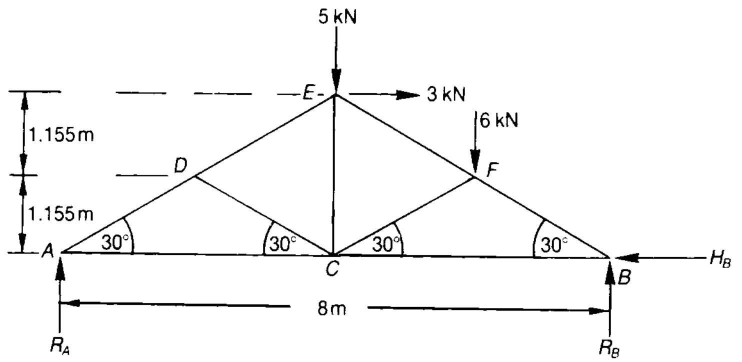


Figure 2.4 Pin-jointed truss.

Solution

Assume all unknown internal forces are in tension, because if they are in compression, their signs will be negative.

As each joint must only have two unknown forces acting on it, it will be necessary to determine the values of R_A, R_B and H_B, prior to using the method of joints.

Resolving the forces horizontally

$$\text{forces to the left} = \text{forces to the right}$$

$$3 = H_B$$

$$\therefore H_B = 3 \text{ kN}$$

Taking moments about B

$$\text{clockwise moments} = \text{counter-clockwise moments}$$

$$R_A \times 8 + 3 \times 2.311 = 5 \times 4 + 6 \times 2$$

$$\therefore R_A = 25.07/8 = 3.13 \text{ kN}$$

Resolving forces vertically

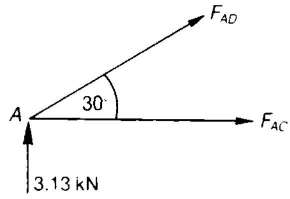
$$\text{upward forces} = \text{downward forces}$$

$$R_A + R_B = 5 + 6$$

or

$$R_B = 11 - 3.13 = 7.87 \text{ kN}$$

Isolate *joint A* and consider equilibrium, as shown by the following free-body diagram.



Resolving forces vertically

$$\text{upward forces} = \text{downward forces}$$

$$3.13 + F_{AD} \sin 30 = 0$$

or
$$F_{AD} = -6.26 \text{ kN (compression)}$$

NB The *negative sign* for this force denotes that this member is in *compression*, and such a member is called a *strut*.

Resolving forces horizontally

$$\text{forces to the right} = \text{forces to the left}$$

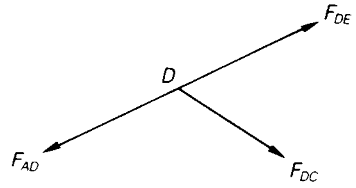
$$F_{AC} + F_{AD} \cos 30 = 0$$

or
$$F_{AC} = 6.26 \times 0.866$$

$$F_{AC} = 5.42 \text{ kN (tension)}$$

NB The *positive sign* for this force denotes that this member if in *tension*, and such a member is called a *tie*.

It is possible now to analyse *joint D*, because F_{AD} is known and therefore the joint has only two unknown forces acting on it, as shown by the free-body diagram.



Resolving vertically

$$\text{upward forces} = \text{downward forces}$$

$$F_{DE} \sin 30 = F_{AD} \sin 30 + F_{DC} \sin 30$$

or
$$F_{DE} = -6.26 + F_{DC} \qquad (2.5)$$

Resolving horizontally

$$\text{forces to the left} = \text{forces to the right}$$

$$F_{AD}\cos 30 = F_{DE}\cos 30 + F_{DC}\cos 30$$

or
$$F_{DE} = -6.26 - F_{DC} \tag{2.6}$$

Equating (2.5) and (2.6)

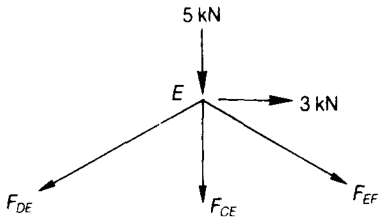
$$-6.26 + F_{DC} = -6.26 - F_{DC}$$

or
$$F_{DC} = 0 \tag{2.7}$$

Substituting equation (2.7) into equation (2.5)

$$F_{DE} = -6.26 \text{ kN (compression)}$$

It is now possible to examine *joint E*, as it has two unknown forces acting on it, as shown:



Resolving horizontally

$$\text{forces to the left} = \text{forces to the right}$$

$$F_{DE}\cos 30 = F_{EF}\cos 30 + 3$$

or
$$F_{EF} = -6.26 - 3/0.866$$

$$F_{EF} = -9.72 \text{ kN (compression)}$$

Resolving vertically

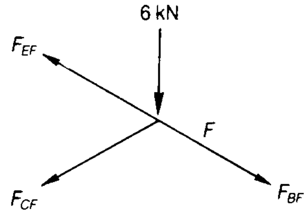
$$\text{upward forces} = \text{downward forces}$$

$$0 = 5 + F_{DE}\sin 30 + F_{CE} + F_{EF}\sin 30$$

$$F_{CE} = -5 + 6.26 \times 0.5 + 9.72 \times 0.5$$

$$F_{CE} = 3 \text{ kN (tension)}$$

It is now possible to analyse either *joint F* or *joint C*, as each of these joints has only got two unknown forces acting on it. Consider *joint F*,



Resolving horizontally

$$\text{forces to the left} = \text{forces to the right}$$

$$F_{EF}\cos 30 + F_{CF}\cos 30 = F_{BF}\cos 30$$

$$\therefore F_{BF} = -9.72 + F_{CF} \tag{2.8}$$

Resolving vertically

$$\text{upward forces} = \text{downward forces}$$

$$F_{EF}\sin 30 = F_{CF}\sin 30 + F_{BF}\sin 30 + 6$$

or

$$F_{BF}\times 0.5 = -9.72\times 0.5 - 0.5\,F_{CF} - 6$$

$$\therefore F_{BF} = -21.72 - F_{CF} \tag{2.9}$$

Equating (2.8) and (2.9)

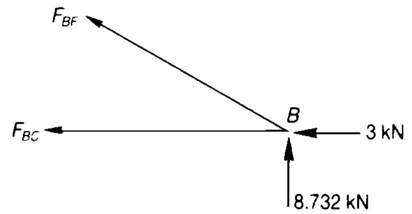
$$-9.72 + F_{CF} = -21.72 - F_{CF}$$

$$\therefore F_{CF} = -6\text{ kN (compression)} \tag{2.10}$$

Substituting equation (2.10) into equation (2.8)

$$F_{BF} = -9.72 - 6 = -15.72\text{ kN (compression)}$$

Consider *joint B* to determine the remaining unknown force, namely F_{BC},



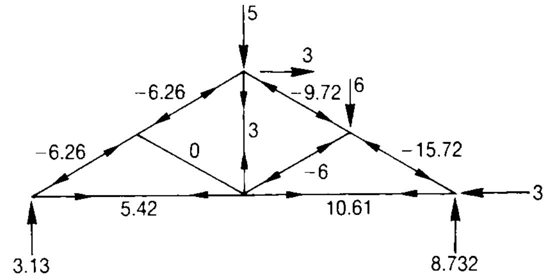
Resolving horizontally

$$\text{forces to the left} = \text{forces to the right}$$

$$F_{BF} \cos 30 + F_{BC} + 3 = 0$$

$$\therefore F_{BC} = -3 + 15.72 \times 0.866 = \text{kN (tension)}$$

Here are the magnitudes and 'directions' of the internal forces in this truss:



2.4 The method of sections

This method is useful if it is required to determine the internal forces in only a few members. The process is to make an imaginary cut across the framework, and then by considering equilibrium, to determine the internal forces in the members that lie across this path. In this method, it is only possible to examine a section that has a maximum of three unknown internal forces, and here again, it is convenient to assume that all unknown forces are in tension.

To demonstrate the method, an imaginary cut will be made through members DE, CD and AC of the truss of Figure 2.4, as shown by the free-body diagram of Figure 2.5

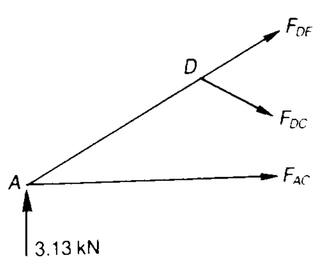


Figure 2.5 Free-body diagram.

Taking moments about D

counter-clockwise couples = clockwise couples

$$F_{AC} \times 1.55 = 3.13 \times 2$$

$$\therefore F_{AC} = 5.42 \text{ kN}$$

NB It was convenient to take moments about *D*, as there were two unknown forces acting through this point and therefore, the arithmetic was simplified.

Resolving vertically

upward forces = downward forces

$$F_{DE} \sin 30 + 3.13 = F_{DC} \sin 30$$

$$\therefore F_{DC} = 6.26 + F_{DE} \tag{2.11}$$

Resolving horizontally

forces to the right = forces to the left

$$F_{DE} \cos 30 + F_{DC} \cos 30 + F_{AC} = 0$$

$$\therefore F_{DC} = -5.42/0.866 - F_{DE}$$

or

$$F_{DC} = -6.26 - F_{DE} \tag{2.12}$$

Equating (2.11) and (2.12)

$$F_{DE} = -6.26 \text{ kN} \tag{2.13}$$

Substituting equation (2.13) into equation (2.11)

$$F_{DC} = 0 \text{ kN}$$

These values can be seen to be the same as those obtained by the method of joints.

2.5 A statically indeterminate problem

In Section 2.1 we mentioned a type of stress analysis problem in which internal stresses are not calculable on considering statical equilibrium alone; such problems are *statically indeterminate*. Consider the rigid beam *BD* of Figure 2.6 which is supported on three wires; suppose the tensions in the wires are T_1, T_2 and T_3. Then by resolving forces vertically, we have

$$T_1 + T_2 + T_3 = 4W \tag{2.14}$$

and by taking moments about the point C, we get

$$T_1 - T_2 - 3T_3 = 0 \tag{2.15}$$

From these equilibrium equations alone we cannot derive the values of the three tensile forces T_1, T_2, T_3; a third equation is found by discussing the extensions of the wires or considering compatibility. If the wires extend by amounts e_1, e_2, e_3, we must have from Figure 2.6(ii) that

$$e_1 + e_3 = 2e_2 \tag{2.16}$$

because the beam BD is rigid. Suppose the wires are all of the same material and cross-sectional area, and that they remain elastic. Then we may write

$$e_1 = \lambda T_1 , \quad e_2 = \lambda T_2 , \quad e_3 = \lambda T_3 , \tag{2.17}$$

where λ is a constant common to the three wires. Then equation (2.16) may be written

$$T_1 + T_3 = 2 T_2 \tag{2.18}$$

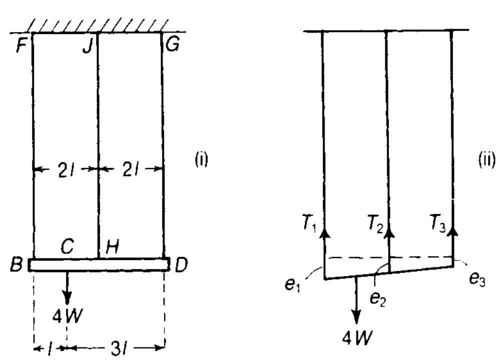


Figure 2.6 A simple statically indeterminate system consisting of a rigid beam supported by three extensible wires.

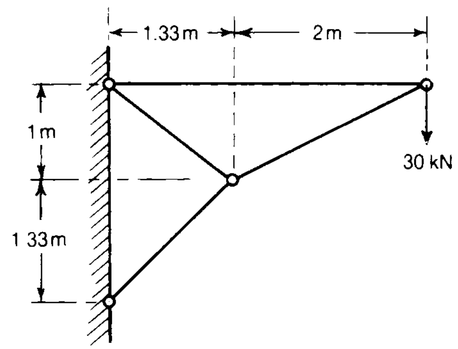
The three equations (2.14), (2.15) and (2.18) then give

$$T_1 = \frac{7W}{12} \quad T_2 = \frac{4W}{12} \quad T_3 = \frac{W}{12} \tag{2.19}$$

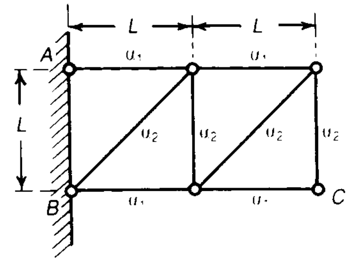
Equation (2.16) is a condition which the extensions of the wires must satisfy; it is called a *strain compatibility* condition. Statically indeterminate problems are soluble if strain compatibilities are considered as well as statical equilibrium.

Further problems *(answers on page 691)*

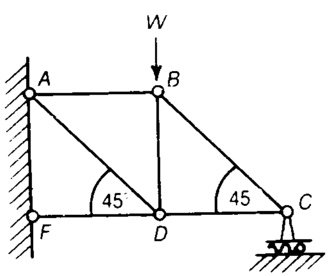
2.2 Determine the internal forces in the plane pin-jointed trusses shown below:



(a)



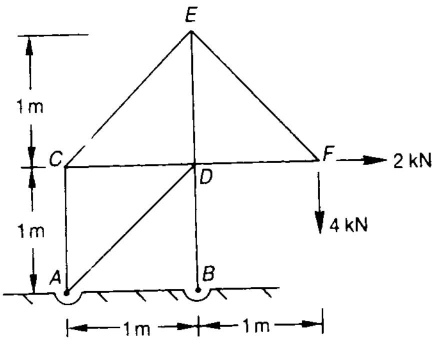
(b)



(c)

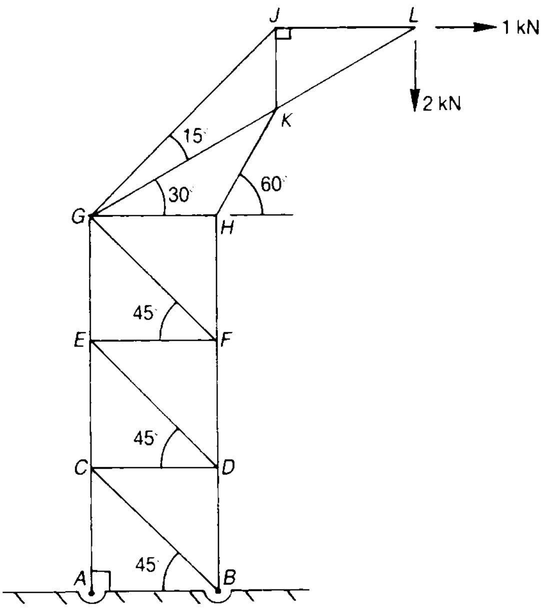
2.3 The plane pin-jointed truss below is firmly pinned at *A* and *B* and subjected to two point loads at the joint *F*.

Using any method, determine the forces in all the members, stating whether they are tensile or compressive. *(Portsmouth 1982)*



2.4 A plane pin-jointed truss is firmly pinned at its base, as shown below.

Determine the forces in the members of this truss, stating whether they are in tension or compression. *(Portsmouth 1980)*



2.5 Determine the internal forces in the pin-jointed truss, below, which is known as a Warren girder.